

# SHAPE MEASUREMENT OF SMALL OPTICAL TARGETS

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Logan, Utah

Contract No. AF19(628)-3825  
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## FINAL REPORT

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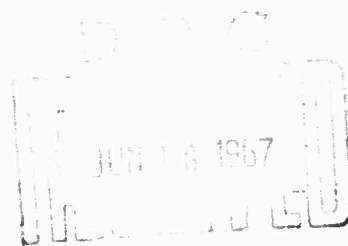
Dr. George A. Vanasse  
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Prepared for  
Air Force Cambridge Research Laboratories  
Office of Aerospace Research  
UNITED STATES AIR FORCE  
Bedford, Massachusetts

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## ABSTRACT

Techniques were developed to determine the shapes of optical objects from images corrupted by noise, diffraction and aberrations. Theoretical and experimental investigations were conducted using these techniques. The major part of the study has been reported in detail in several Scientific Reports but is reviewed here, along with the results of some preliminary and subsidiary work.

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## OBJECTIVES

The overall objectives of this investigation were the derivation of effective optical methods for the determination of the shapes of objects and the evaluation of these methods. Initially, attention was to be confined to a few standard shapes such as circles, rectangles, and triangles. The basic limitations imposed on system design and performance by diffraction and noise were to be investigated both theoretically and experimentally, as well as the effects of such perturbations as non-uniform illumination, aberrations, and film granularity. These objectives were essentially adhered to, with the main emphasis being placed upon theoretical investigations.

## INTRODUCTION

This final report summarizes the work accomplished under Air Force Cambridge Research Laboratories Contract No. AF19(628)-3825. This study was done under Project No. 8663 of the Advanced Research Projects Agency and under Project ED-30 of the Utah State University Electrodynamics Laboratories. A list of scientific reports and publications resulting from study under this contract are found at the end of this report.

## TECHNICAL RESULTS

The first part of this report describes some of the preliminary studies that were carried out to determine the effects of object illumination and diffraction on the ability to discriminate between simple objects. The second part briefly describes the results of the basic work reported on in the interim Scientific Reports that resulted from this contract. For the detailed results, reference should be made to the Scientific Reports.

### Preliminary Study

#### Diffraction and Fourier analysis

It has been shown that if attention is limited to Fraunhofer diffraction, the techniques of Fourier analysis can be used to investigate the performance of an optical system. The system can then be treated as a linear filter of spatial frequencies whose properties are described by a transfer function analogous to that of an electrical filter. For incoherent illumination, the spatial-frequency spectrum of the image is found by multiplying the spatial-frequency spectrum of the object by the system transfer function.

Alternately, the image intensity distribution may be viewed as the convolution of the object intensity distribution with the point spread function of the optical system. The point spread function is simply the



image intensity distribution due to a point source and is analogous to the impulse response of an electrical filter. It is the Fourier transform of the system transfer function.

By viewing the image intensity distribution as a convolution, one can quickly obtain a qualitative understanding of the effects of diffraction. For a circular aperture, the point spread function is the well-known Airy pattern of the form

$$\left[ 2 \frac{J_1(\alpha r)}{\alpha r} \right]^2$$

This pattern has a central maximum whose width is inversely proportional to the aperture radius, together with fringes or sidelobes which decrease rapidly in strength as  $r$  increases. Convolution with this pattern produces an image which is a smeared version of the object. The amount of smearing increases, of course, as the width of the Airy pattern increases. Hence, if the object (represented in the appropriate coordinate system) is very large compared with the Airy disc, the smearing produced by diffraction will be negligible.

As the size of the object relative to the size of the Airy disc is reduced, smearing becomes more important. For example, the corners of a square become more and more rounded, and it becomes increasingly more difficult to discriminate between a square and a circle. As the object sizes continue to be reduced, a condition will be reached at which

discrimination is not sufficiently reliable. This limit will depend on the patterns to be classified, the method of classification or recognition, the characteristics of the noise in the system, and the degree of reliability required.

### Theoretical imaging and discrimination

As was pointed out earlier, the spatial-frequency spectrum of the output (image) can be written as the product of the spatial-frequency spectrum of the input (object) and an appropriately defined system transfer function. Alternately, the image intensity distribution may be viewed as the convolution of the object intensity distribution with the point spread function of the system. These two characterizations provide two alternate methods of studying the effects of the optical system upon the object. For some purposes, it is more convenient to work in the spatial-frequency domain, while in other cases the intensity-distribution domain is more convenient.

These techniques can be applied to the problem of determining the effects due to diffraction by a finite circular aperture. In order to isolate the effects of diffraction from other sources of perturbation, the problem should be formulated in as simple a form as possible. Specifically, the problem considered could be the following. Assume that one of a finite number of exactly known objects (e.g., triangle, square, circle) is present at the input to an optical system. In addition to the object, a

certain amount of background radiation is present. The intensity distribution in the image plane is thus a perturbed and filtered version of the original object. It is desired to observe this image-plane intensity distribution and to determine which object is present at the system input.

An efficient decision scheme can be devised by using the techniques of statistical decision theory. The problem can be considered as that of deciding which member of a set of known (or partially known) signals has been received. Using the techniques of statistical decision theory, the optimum or near-optimum extraction or classification procedures can be derived for various noise environments. In addition, if the model is sufficiently simple, the performance of these procedures can be evaluated according to a criterion such as the probability of misclassification. Within this framework, the effects of diffraction, aberrations, nonuniform illumination, film granularity, and perhaps other phenomena on the performances of various classification procedures can be investigated.

Under certain reasonable assumptions, it can be shown that the optimum decision scheme uses correlation techniques on matched filters. Even when they are not optimum, such techniques have been shown to give good results. It is important to note that direct application of these techniques requires that the patterns to be classified are known exactly. When this assumption is relaxed, the decision procedure must be modified accordingly.

One problem which has not been mentioned thus far is that of filter mismatch (i. e., matching a filter to the wrong signal). This problem will be important for two reasons. First, if we match the filter to the object itself (e. g., a circle), it will not be matched to the image, since the image has been affected by the finite aperture. In principle, we should match to the intensity distribution which results when the object intensity distribution is convolved with the system point spread function. To do this exactly, however, is often impractical; hence, our filter will usually be at best slightly mismatched.

A second potential cause of filter mismatch is nonuniform illumination of the object. If the distribution of intensity across the object is exactly known to us, we can again, in principle, achieve a filter which is exactly matched. In practice, there will usually be some mismatch due to our lack of precise knowledge concerning the object illumination.

The foregoing discussion indicates the necessity for considering the effects of filter mismatch in evaluating the performance of a matched-filter classification system. These effects are included in Scientific Report No. 4.

It should be emphasized again that the work described above presupposes precise knowledge of the shape, size, and position of all the patterns to be classified. This is certainly a reasonable starting point, because it tends to remove any extraneous sources of uncertainty, thereby

allowing us to concentrate on the effects of diffraction. In most practical situations, however, such precise knowledge regarding the patterns will not be available. Hence, it will be necessary to modify the decision procedures accordingly. An additional complicating factor is the fact that we will often be interested in automatic pattern classification by machine. Hence, we seek techniques which utilize the incomplete information available in a way which can be easily automated. In many cases, the only information available about the patterns to be classified will be samples of these patterns. This fact leads to a consideration of machine learning or adaptation.

#### Preliminary experimental imaging and discrimination

Sources with uniform and non-uniform illumination were constructed to experimentally determine the effects of such illumination on the ability to classify patterns. The results for various geometrical and illumination parameters are reported here.

#### Uniform illumination

A uniformly-illuminated set of patterns (triangles, squares, and circles) on a black background was photographed with the lens 25 feet from the patterns. The size of each of the patterns is given in Table 1.

Table 1. Image pattern size.

Pattern No.	Square Side (in.)	Triangle Altitude (in.)	Circle Diameter (in.)	Angle Subtended (Rad.)
1	0.0625	0.0625	0.0625	$2.08 \times 10^{-4}$
2	0.125	0.125	0.125	$4.16 \times 10^{-4}$
3	0.250	0.250	0.250	$8.3 \times 10^{-4}$
4	0.500	0.500	0.500	$16.7 \times 10^{-4}$

The angle subtended by the pattern is found by dividing the distance across the patterns by the distance from the pattern to the lens. The size of the Airy disc is given by

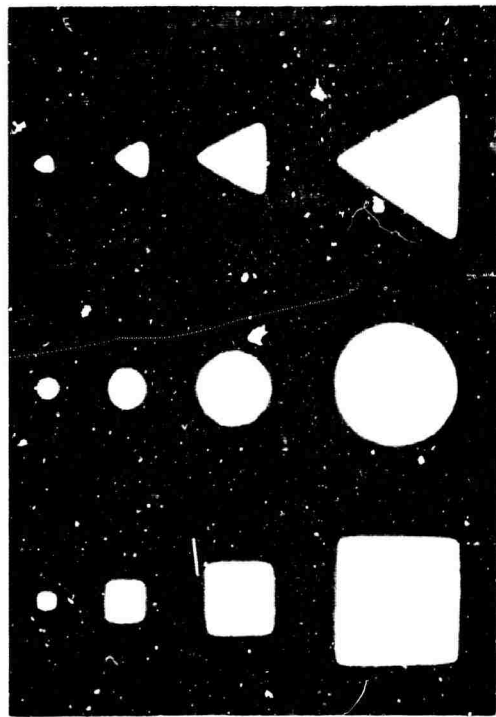
$$\theta = 1.22 \lambda / d$$

where  $\lambda$  is the wavelength of light and  $d$  is the diameter of the lens aperture. The circular apertures and the corresponding Airy disc size are listed in Table 2.

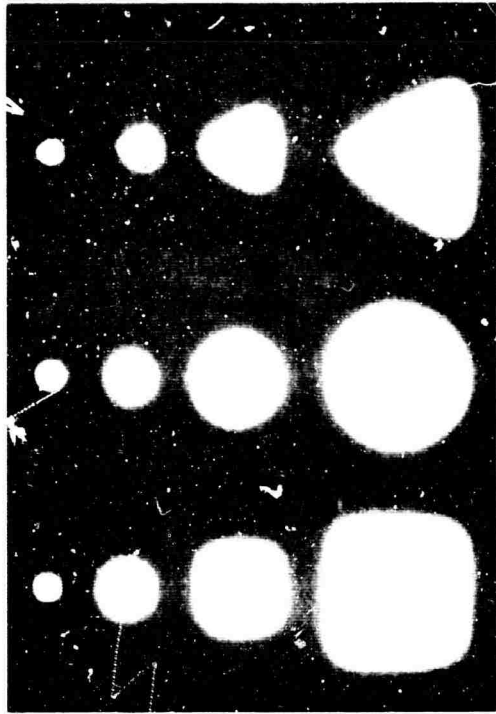
Table 2. Airy disc for various circular apertures.

Aperture No.	Size of Aperture Diameter (in.)	Airy Disc (radians)
1	0.640	$3.77 \times 10^{-5}$
2	0.484	$4.97 \times 10^{-5}$
3	0.323	$7.44 \times 10^{-5}$
4	0.242	$9.94 \times 10^{-5}$
5	0.173	$1.39 \times 10^{-4}$
6	0.120	$2.01 \times 10^{-4}$
7	0.081	$3.00 \times 10^{-4}$
8	0.060	$4.02 \times 10^{-4}$
9	0.040	$6.02 \times 10^{-4}$
10	0.028	$8.59 \times 10^{-4}$
11	0.020	$1.20 \times 10^{-3}$
12	0.014	$1.78 \times 10^{-3}$

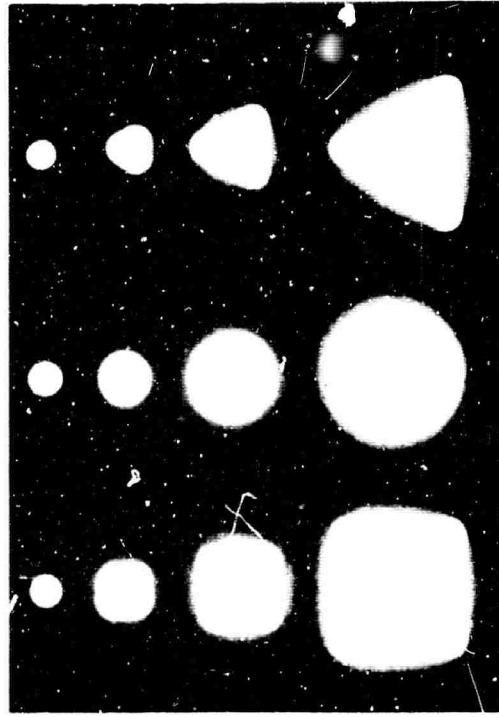
Figure 1 shows a portion of the results of photographing each pattern after having been diffracted by the circular apertures listed in Table 2. The photograph taken with aperture No. 6 (Airy disc  $2.01 \times 10^{-4}$  radians, pattern angle subtended  $2.08 \times 10^{-4}$  radians) is the first photograph in which the smallest patterns are visually indistinguishable. For aperture No. 8, the second row of patterns (Airy disc  $4.02 \times 10^{-4}$ , pattern angle



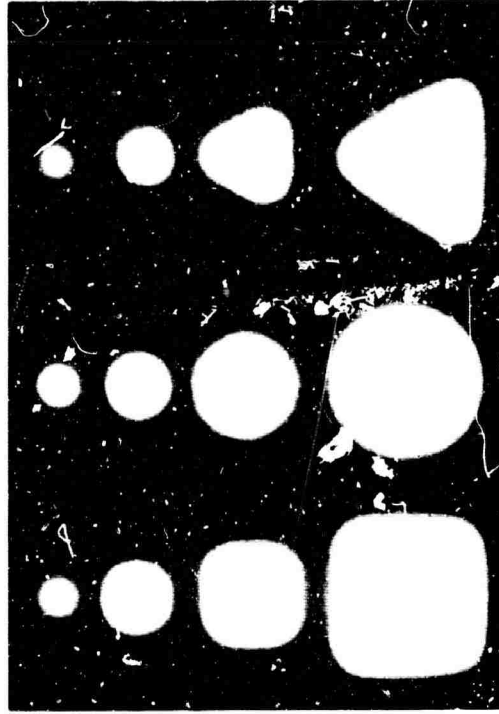
Aperture No. 1



Aperture No. 5



Aperture No. 6



Aperture No. 8

Figure 1. Photographs showing diffraction limiting of small objects.



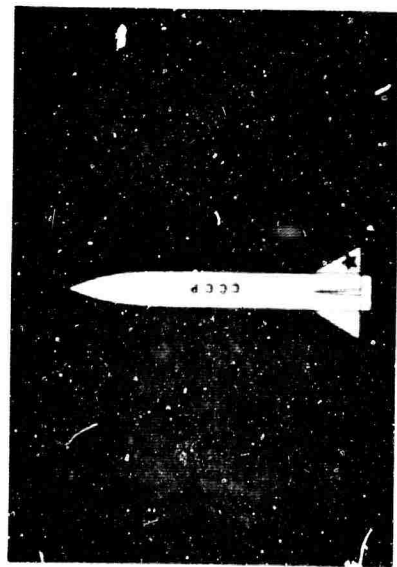
subtended  $4.16 \times 10^{-4}$  radians) approaches the limit at which the geometrical shapes can be visually distinguished. The crucial relationship shown by these photographs is this: When the ratio of the Airy disc size to pattern angle subtended is unity (or greater), then visual recognition of the various patterns is not reliable. This relationship is discussed further in Scientific Report Nos. 8 and 9.

Visual diffraction limiting of a uniformly illuminated rocket pattern is shown in the photographs of Figure 2. Notice that the tail structure (shape of the fins) is visually obscured when the ratio of the Airy disc and the smaller dimensions of the rocket ( $7.29 \times 10^{-4}$  radians) approaches unity. In the last photograph (aperture No. 12) even the gross longitudinal features of the rocket are obscured.

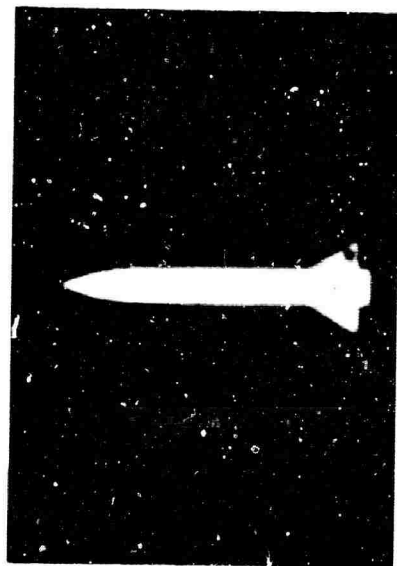
#### Nonuniform illumination

The visual obscuration effects of diffraction can be even more severe when the objects to be distinguished are nonuniformly illuminated. To obtain a qualitative visual estimate of the effects of nonuniform illumination, two nonuniformly illuminated square sources were diffracted by various sizes of circular apertures.

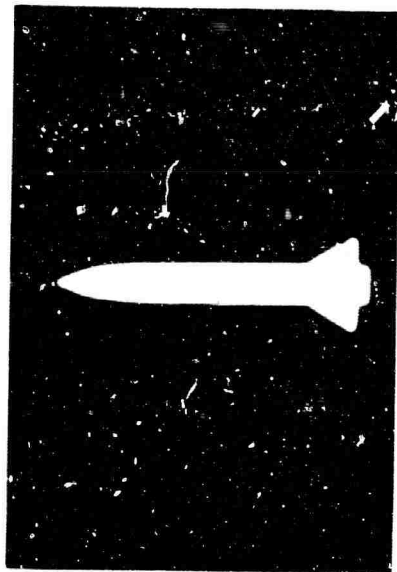
The optical system used is shown in Figure 3. The lenses  $L_1$  and  $L_2$  serve to reduce the distance between the object plane and image plane over the system previously used for the uniformly illuminated sources. The



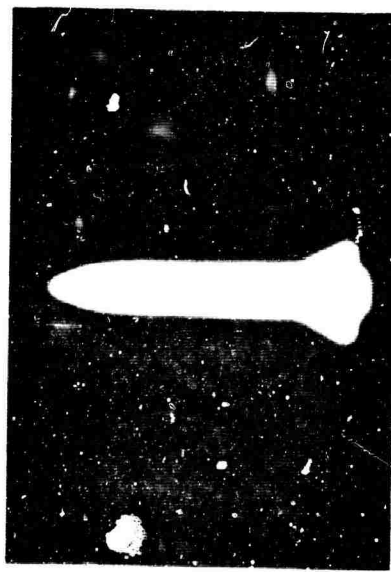
Aperture No. 1  
(airy disk size  $3.77 \times 10^{-5}$  radians)



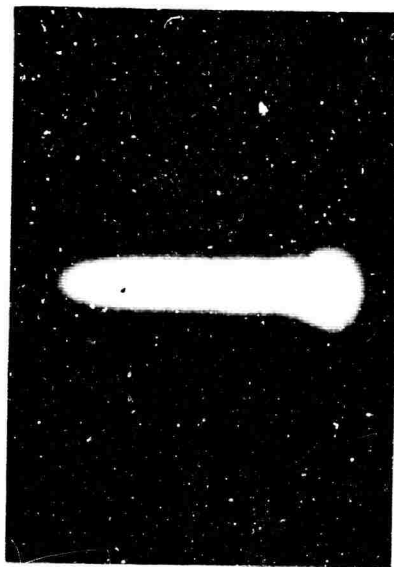
Aperture No. 4  
(airy disk size  $9.94 \times 10^{-5}$  radians)



Aperture No. 6  
(airy disk size  $2.01 \times 10^{-4}$  radians)



Aperture No. 8  
(airy disk size  $4.02 \times 10^{-4}$  radians)



Aperture No. 10  
(airy disk size  $8.59 \times 10^{-4}$  radians)



Aperture No. 12  
(airy disk size  $1.78 \times 10^{-3}$  radians)

Figure 2. Photographs showing diffraction limiting of a rocket pattern (angle subtended by rocket pattern:  $6.04 \times 10^{-3}$  by  $7.29 \times 10^{-4}$  radians).

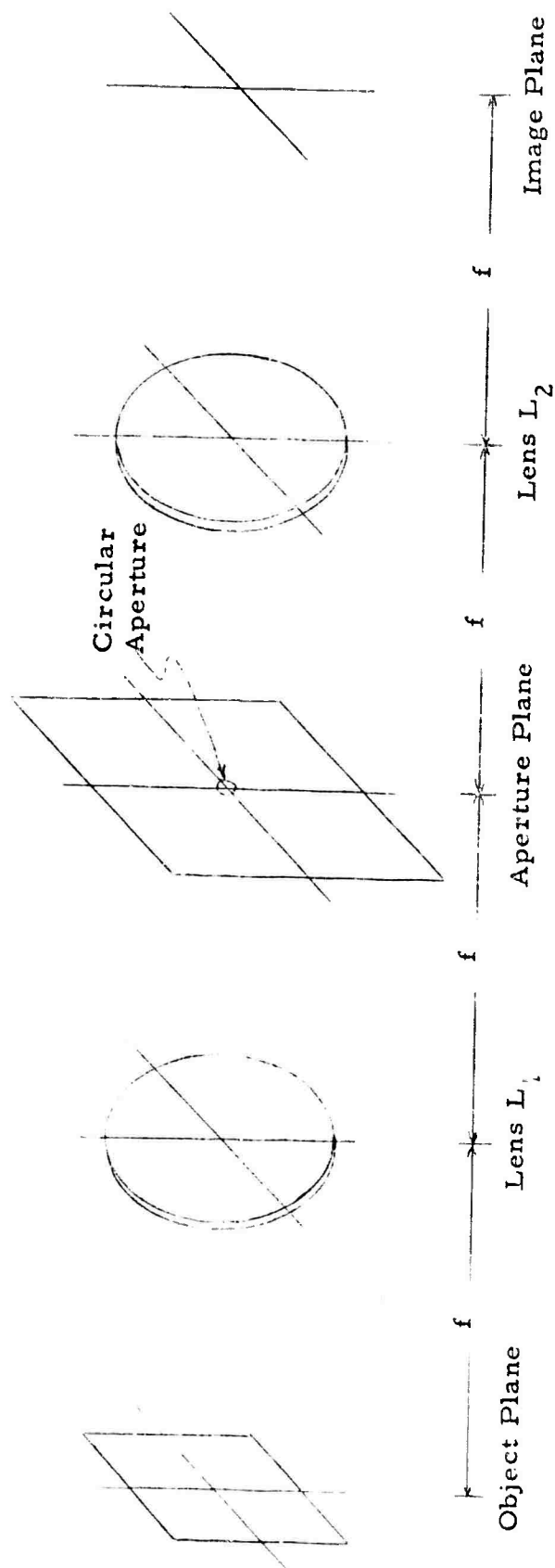


Figure 3. Optical system used to illustrate the diffraction effects on nonuniformly illuminated square sources.

Fourier techniques referred to previously are valid for both systems. The two sources were made from negatives of nonuniformly exposed photographic film and then illuminated from behind to produce the desired nonuniform sources. Figure 4 shows microdensitometer tracings of the sources used. The nonuniformity is clearly evident and shows a linear variation of approximately 100:1 in contrast.

Figure 5 presents a few of the photographs obtained with this system. The top two photographs 5(a) and 5(b) are the sources imaged through the optical system when no appreciable diffraction is present (No. 7 aperture). Figures 5(c) and 5(d) show the results when more diffraction is present (aperture No. 9). The exposure times for photographs 5(c) and 5(d) were identical. Increasing the diffraction still further (aperture No. 10) produced the photographs 5(e) and 5(f). The exposure times for 5(e) and 5(f) were identical.

A visual comparison of 5(c) with 5(d) and 5(e) with 5(f) shows that the diffraction effects are definitely more severe when the corners of the square are less brightly illuminated. A very rough (but informative) quantitative statement of the effects of nonuniform illumination can be made as follows. Suppose we decide visually that the diffraction imposed by aperture No. 9 is sufficient to obscure the object shape for the photograph in 5(c), but the object shape for the series 5(b), 5(d) and 5(f) is not obscured until aperture No. 10 is used in 5(f). Such a visual decision

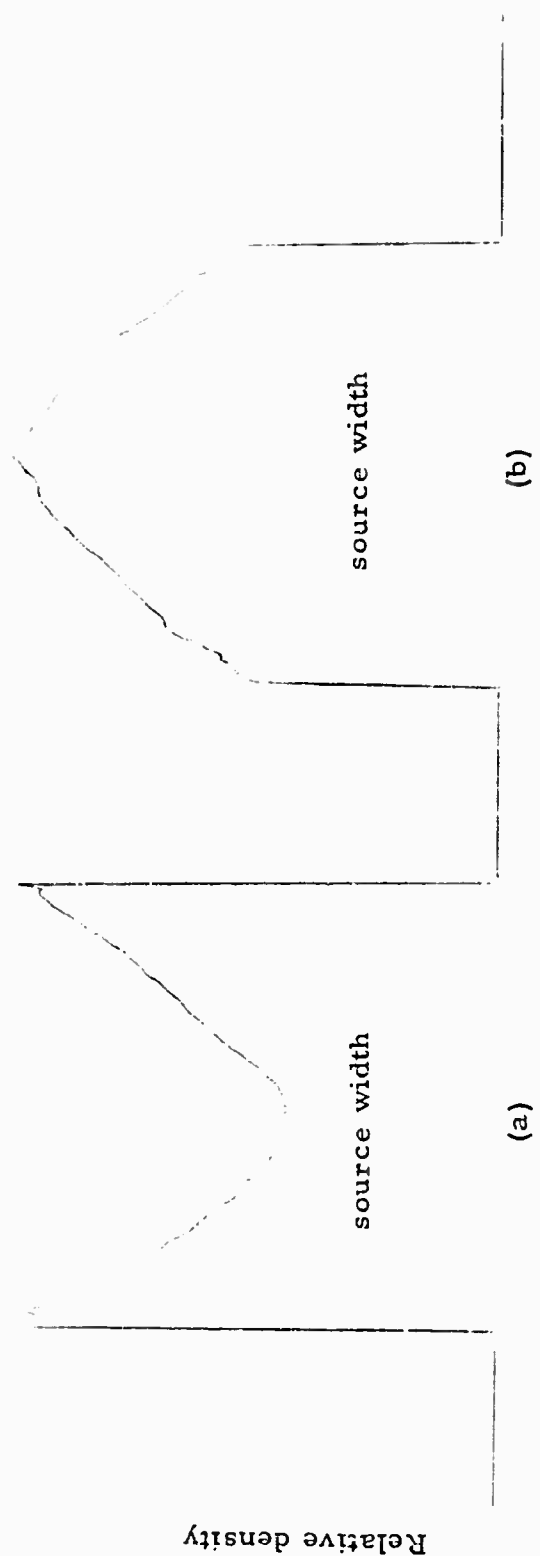


Figure 4. Microdensitometer tracings showing the nonuniformity in illumination for the two sources used. Tracings (a) and (b) correspond, respectively, to the sources that produced the undiffracted images of Figures 5(a) and 5(b). (The relative density of the tracings is inversely related to light illumination.)

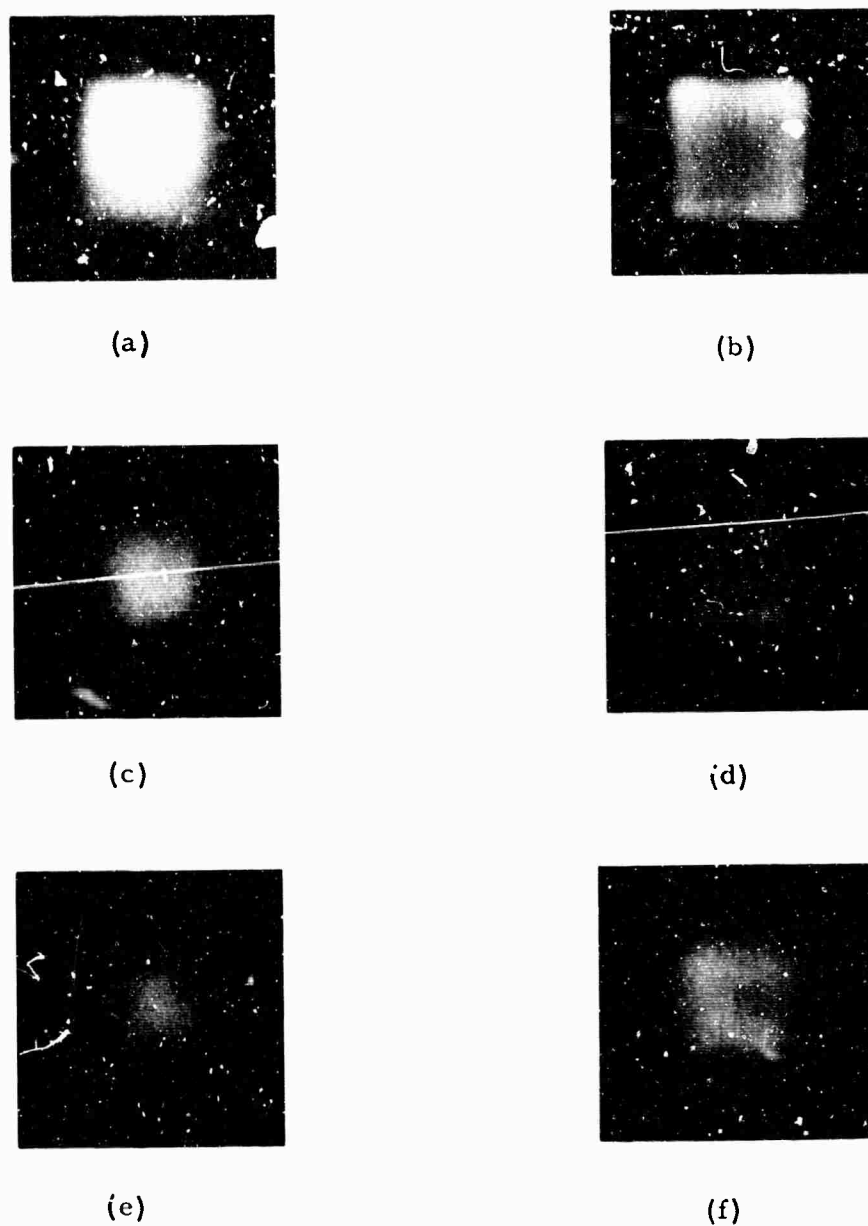


Figure 5. Photographs (a) and (b) show two nonuniformly illuminated objects imaged through the optical system when no appreciable diffraction is present. Photographs (c) and (e) are diffracted (aperture Nos. 7 and 10, respectively) versions of (a). Photographs (d) and (f) are diffracted (aperture Nos. 9 and 10, respectively) versions of (b).

allows one to state that the nonuniformly illuminated source with bright corners can withstand 40% more diffraction than the source with dim corners. This statement, of course, depends upon the dynamic range (relative contrast in intensity) and distribution of the nonuniformity of the source illumination. Further quantitative statements in mathematical terms are presented in Scientific Report No. 4.

To summarize, it is noted that: (1) When the objects are uniformly illuminated, diffraction sufficient to visually obscure object shape occurs when the ratio of the Airy disc to the object size is unity or greater; (2) When details of the object are less brightly (nonuniformly) illuminated, the visual diffraction limit occurs when the above ratio is less than unity. To visually discriminate between various objects beyond these limits will in general require restoration procedures (Scientific Report No. 8) to be performed.

## SUMMARY OF SCIENTIFIC REPORTS GENERATED UNDER THIS CONTRACT

An important problem in optical data processing is that of determining whether or not a particular pattern or signal is present in the image plane of an optical system. Various versions of this problem can be obtained, depending on the prior knowledge available about the pattern or signal to be detected and the noise in the system. The methods of statistical decision theory can be used to derive optimum detection procedures which depend on this prior information and on the costs of the different types of errors.

The problem of recognizing patterns in photographs was considered [1]. There are many important problems which require recognition or classification of optical patterns in the presence of background radiation. In many such problems, the data to be analyzed consist of one or more photographs of the noisy pattern which is to be classified. The accuracy of any procedure for classifying patterns in photographs will, in general, be limited by such factors as atmospheric distortion, lens diffraction and aberration, and film granularity, as well as by the ambient background radiation.

For the specific case considered, the patterns to be classified were assumed to be exactly known immediately prior to their impinging upon the film. Making this assumption amounts to ignoring atmospheric distortion



and assuming that, if diffraction and aberration are important, their effects on the pattern are known. The photographs were assumed to be sampled and quantized to form a binary vector (i.e., the elements of the vector consist of ones and zeros depending, respectively, upon whether the quantized pattern elements are present or absent). It was found that the optimum classification procedure performs nonlinear operations on the received signal vectors. For uniform pattern illumination or low signal-to-noise ratio, the optimum procedure reduced to one using matched-filter techniques. The analysis applies not only to the specific problem mentioned here but can be applied to any situation in which the patterns to be classified can be represented by binary vectors.

Another problem considered, which is similar to the one above, was the problem of detecting an optical pattern whose shape and orientation are known but whose position is unknown [2]. Two detection procedures were investigated for this case--one which entailed a search procedure and one which did not. The false-alarm and false-dismissal probabilities (i.e., respectively, the probability of saying the "signal" is present when it is absent and the probability of saying the "signal" is absent when it is present) for these two procedures were evaluated under certain simplifying assumptions in order to compare the two procedures with each other and with the detector which is optimum when the position of the pattern is known. It was shown that there is a trade-off between error rate and

information rate. The procedure requiring a search technique processes the data less rapidly but at the same time achieves a lower error rate for a given signal-to-noise ratio. The choice of procedure will depend on the signal and noise parameters and on the performance required. This analysis can also be applied to the problem of detecting a signal with unknown arrival time.

The effects of diffraction and additive noise on the ability to discriminate between two known optical intensity distributions were investigated [4]. Specifically, the problem of discriminating between an incoherently illuminated circular source and an incoherently illuminated rectangular source, both of which had been diffracted by a circular aperture, was considered. While this problem may be somewhat artificial, it nevertheless provides considerable insight into the ultimate limitations on system performance due to diffraction and noise.

In order to obtain quantitative results pertaining to the design and evaluation of a discrimination procedure, it was necessary to specify a particular noise model. The model used was based on the assumption that the intensity distribution in the image plane impinges on a photosensitive detector whose output consists of photoelectrons which obey a Poisson distribution. The model was discussed in some detail, and the associated optimum discrimination procedure was obtained. Because this optimum procedure would be hard to implement, some important sub-optimum procedures were also considered. It was shown that a key quantity in

assessing the effect of diffraction on our ability to discriminate between two different objects is the integrated square of the difference between the images produced by the two objects. This quantity is an important performance criterion in its own right, and the results pertaining to it are not tied to the particular noise model mentioned above. The results of the evaluation of the discrimination procedures were in the form of integrals which were not easily evaluated by analytical means. These integrals were evaluated numerically, and the results were presented and discussed. Typically, it was shown that the ability to discriminate drops off quite rapidly as the size of the Airy disc approaches the size of the object.

The detectors devised for the above studies were fixed-sample detectors, that is, detectors for which the number of samples or sample duration is fixed in advance. Another fixed-sample detection problem was studied for which the objective was to discriminate between optical signals which had been distorted by diffraction, additive background noise, and multiplicative noise [8]. For this study Poisson statistics were assumed, and the observed output vectors consisted of the number of photoelectrons. The structure and performance of fixed-sample detectors designed for random or unknown objects (optical signals) as well as known objects were investigated. Optimum procedures, in the sense of minimum risk, were derived for detecting known and unknown objects.

The study of the performance of these detection procedures illustrated the effect of uncertainty about the object.

A basic and important problem in optical data processing is to determine in as short a time as possible the presence or absence of an object in the field of view of an optical instrument. When time is important, as it is in many situations, one is led to consider sequential data processing procedures instead of conventional fixed-sample procedures. In a sequential detection procedure, the number of samples is determined by the course of the experiment. If the received data are relatively noise-free, a reliable decision may be made quickly, while for very noisy data the test may last considerably longer. The sequential detection test has a shorter average test length than the fixed-sample test of the same accuracy.

A study was made of the problem of sequentially detecting a signal whose amplitude was fixed but initially unknown to the system designer [6]. Under the assumption that the signal was drawn from a Gaussian population with known parameters, the sequential likelihood-ratio detector for detecting the signal in the presence of Gaussian noise was derived. The amount of information per sample provided by this detector was calculated and compared with that for a detector designed for a specific signal strength. This quantity provides considerable insight into the adaptive capability and the performance of the detector. The average sample number (ASN) necessary to make a decision with a prescribed

error probability was investigated using approximate analytical techniques in conjunction with computer simulation. This detector was compared with a sequential detector designed for a specific signal strength and was shown to achieve considerably greater protection against making decision errors for the presence of small signals at the cost of an increased average test length. An approximate analysis was shown to be in good agreement with results obtained by computer simulation.

Sequential detection was also considered for the more realistic case in which the noise and signals obey Poisson statistics [8]. Sequential detectors were derived for various cases of known and unknown optical signals and their performances were compared. The information per sample and ASN were determined and compared for these various cases. For most of the cases considered, analytical solutions were extremely difficult to obtain; hence, the analysis was, to a large extent, carried out by computer simulation.

In the detection or discrimination studies discussed above, we were concerned with the problem of determining which of a finite number of states of nature (usually two) was the correct one. The problem of estimation is closely related to that of detection. The difference between the two problems is that in the estimation problem the possible states of nature form a continuum, whereas in the detection problem they form a discrete set. Thus, the estimation problem may be thought of as a

detection problem with a continuum of possible states. A large part of the study effort dealt with the problem of estimating incoherent optical signals.

The problem of estimating the intensity distribution of an optical signal which had been distorted by diffraction and background noise was studied. Under the assumption that the signal and noise were additive, the optimum linear estimate of the intensity distribution, using the criterion of minimum mean-square error, was obtained [3]. This estimate was shown to be a linearly-filtered combination of the a priori mean and the observed image distribution, with the details of the filtering depending on the prior information, the noise statistics, and the optical imaging system. The performance of the optimum estimation procedure was evaluated for the important special case of large a priori uncertainty. The optimum procedure for processing multiple observations was also considered. It was shown that as the number of observations became large, the estimate approached the true object.

A further study was made of the estimation problem for the presence of multiplicative noise (detection noise) as well as additive background noise and diffraction [8]. Poisson statistics were assumed in this study, since in many real situations Poisson statistics are more realistic than the more commonly used Gaussian statistics. The performance of the minimum mean-square-error estimation procedure was evaluated for several special cases. Some results pertinent to optimum sampling

schemes were obtained for both white and colored additive noise. Other types of estimates were also briefly considered.

A basically numerical approach was applied to the problem of estimating or restoring optical signals which are obscured by diffraction and noise [7]. The numerical approach used was to operate directly on the noisy image and point spread function (impulse response of the optical system) rather than on the Fourier transform of these quantities. Special emphasis was placed on studying the effect of noise and the use of a priori information in the restoration process. Several types of "optimum" estimates of the object intensity distribution were considered. These statistical estimates were compared mathematically, and in many cases numerically, to other non-statistical estimates formulated from control theory and dynamic programming. Extensive numerical results were obtained for the restoration of various one-dimensional objects in the presence of noise. Two monochromatic "point sources" in the presence of noise were shown to be resolved when separated by  $1/5$  of the Rayleigh criterion distance. Numerical results were also shown for the mean-square error as a function of a priori information, the measuring scheme chosen, and diffraction. Using these results, and having been given an estimate of the noise variance and diffraction, one can to some extent ascertain the detail and resolution expected in the restoration process.

A somewhat different study from those mentioned above was an experimental investigation to develop a procedure whereby the Michelson stellar interferometer could be used to spatially analyze an unknown quasi-monochromatic source [5]. It was found from this investigation that a uniformly illuminated symmetrical source, located far enough from the analyzing interferometer to satisfy Fraunhofer conditions, can be analyzed spatially to determine the absolute size of the source (e.g., source width for the case of a one-dimensional slit source). A continuously variable double slit arrangement was developed and used to uniformly increase the slit separation which was necessary to Fourier analyze the source. The envelope of the intensity curve obtained by placing a fixed detector in the region of the first order of diffraction while the slits were uniformly separated was found to be the visibility curve. The Fourier transform of this curve has been shown to be the intensity distribution of the source.

A study was also made of a real-time Fourier transform technique [7]. It has been found that the spectral distribution of radiation can be obtained by calculating the Fourier transform of the interferogram obtained from a two-beam interferometer. The use of a digital computer has the disadvantage that the spectrum is not available for immediate analysis. The study of a real-time Fourier transform was undertaken to determine the feasibility of computing spectra in real time and building a computer to perform this task.



Various computational techniques for performing the required transformation were studied and a real-time Fourier transform synthesizer was devised. It was designed to be compatible with a lamellar grating interferometer, but it could be used just as well with a well-compensated Michelson interferometer. A novel approach was used to obtain the cosine functions required in the synthesis of the cosine transform. All of the computations except the summation were performed in analog form, while the summation was performed digitally. The result of the computation, the spectral distribution, is available as soon as the interferogram is obtained. Errors in the technique were investigated and methods of compensating for them determined. Data were obtained for several test functions, and the synthesized spectra were compared with the theoretical values. The results of the spectral synthesis were obtained for several actual interferograms.

Scientific Reports Generated under Contract No. AF19(628)-3825

1. Rushforth, C.K., Recognizing Patterns in Photographs. Scientific Report No. 1, AFCRL-64-640, Contract No. AF19(628)-3825, Electro-Dynamics Laboratories, Utah State University, Logan, Utah, 1 May 1964.
2. Rushforth, C.K., Detection of a Pattern in Unknown Position. Scientific Report No. 2, AFCRL-64-1011, Contract No. AF19(628)-3825, Electro-Dynamics Laboratories, Utah State University, Logan, Utah, 1 November 1964.
3. Rushforth, C.K., Restoration of Optical Patterns. Scientific Report No. 3, AFCRL-65-181, Contract No. AF19(628)-3825, Electro-Dynamics Laboratories, Utah State University Logan, Utah, 1 March 1965. (Also presented at Opt. Soc. Am. Spring Meeting, 30 March 1965.)
4. Rushforth, C.K., and R.W. Harris, The Effects of Diffraction on Optical Discrimination. Scientific Report No. 4, AFCRL-66-105, Contract No. AF19(628)-3825, Electro-Dynamics Laboratories, Utah State University, Logan, Utah, 1 January 1966.
5. Allen, Royce H., A Continuously Variable Double Slit System and the Fourier Analysis of Michelson's Stellar Interferometer. Scientific Report No. 5, AFCRL-66-399, Contract No. AF19(628)-3825, Electro-Dynamics Laboratories, Utah State University, Logan, Utah, 1 May 1966.
6. Rushforth, C.K., Adaptive Sequential Detection of an Unknown Signal. Scientific Report No. 6, AFCRL-66-491, Contract AF19(628)-3825, Electro-Dynamics Laboratories, Utah State University, Logan, Utah, 1 June 1966.
7. Hoffman, Joseph E., Jr., A Real Time Fourier Transform Synthesizer. Scientific Report No. 7, AFCRL-67-0049, Contract No. AF19(628)-3825, Electro-Dynamics Laboratories, Utah State University, Logan, Utah, 1 November 1966.

8. Harris, R. W., and C. K. Rushforth, Numerical Restoration of Optical Objects Obscured by Diffraction and Noise. Scientific Report No. 8, AFCRL-67-0070, Contract No. AF19(628)-3325, Electro-Dynamics Laboratories, Utah State University, Logan, Utah, 1 December 1966.
9. Austin, M. C., and C. K. Rushforth, Estimation and Detection of Optical Signals Distorted by Diffraction, Background Noise, and Detection Noise. Scientific Report No. 9, AFCRL-67-0071, Contract No. AF19(628)-3325, Electro-Dynamics Laboratories, Utah State University, Logan, Utah, 1 December 1966.

Publications in Professional Journals Generated to Date  
under Contract No. AF19(628)-3825

Hoffman, Joseph E., and George A. Vanasse. 1966. Real-Time Spectral Synthesis in Fourier Spectroscopy, Applied Optics 5: 1167-1169.

Rushforth, Craig H. 1965. Recognizing Patterns in Photographs, Applied Optics 4:379-381.

Wood, John K., and Royce H. Allen. 1966. A Double Slit with Continuously Variable Separation, Applied Optics 5:351.

## RECOMMENDATIONS FOR FUTURE STUDY

It has been shown (Scientific Report No. 8) that the restoration of diffracted optical objects is feasible for SNR's which are possible to achieve in practice. Although the procedure was specifically developed for and applied to the optics problem, resolution enhancement is obviously applicable to other areas, such as spectroscopy and radar. In interferometry, the basic resolution limitation in the power spectrum is due to a finite path length difference obtainable in the interferometer. Provided the SNR is high enough, the restoration procedure can effectively extend the path length difference (mathematically) and thus overcome the physical limitations in the interferometer. In a particular example, the resolution has been shown to be doubled when the SNR is as low as 30. We feel that these results clearly imply that further investigation of the application of resolution enhancement is both necessary and warranted.

The results of this study (Scientific Report No. 4, Scientific Report No. 8) indicate that it is more difficult and costly (in terms of SNR) to restore the entire optical object in every detail than to restore a smaller set of parameters. For example, for a specific application in the interferometry case, perhaps only parameters such as the number of spectral lines in a given region, the spacing between the lines, the line

widths, or the relative magnitude of the lines with respect to a common reference may be necessary. It appears that these parameters could be estimated less expensively in terms of SNR than obtaining the entire spectra. In view of these results, it is recommended that a study be made which will put on an absolute scale the trade-off between SNR and the error involved in estimating smaller sets of parameters such as the set mentioned in the example above.

It is also recommended that the concept of a discrete sequential detector, described in Scientific Report No. 9, be extended to that for a continuous sequential detector where only one observation is made but the duration of the observation is the variable. Also, multiple detection (fixed-sample and sequential) should be considered for cases with more than two states of nature. That is, a discrimination procedure might consist of observing the output vector of the detector and deciding which of  $M$  possible optical patterns is present. The results of this additional study as well as the results of Scientific Reports No. 5 and No. 9, could be used in interferometric spectroscopy for the discrimination of spectra.

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14. KEY WORDS	LINK A		LINK B		LINK C	
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